NONISOTHERMAL FLOW OF ANOMALOUSLY VISCOUS LIQUIDS IN THE CHANNELS OF SCREW EXTRUDERS

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The nonisothermal flow of an anomalously viscous liquid in the channel of a screw extruder is analyzed with allowance for the influence of the side walls. A comparison with experimental data is given.

A rather large number of reports have been devoted to the investigation of the laws of motion and heat exchange in processes of polymer treatment using screw extruders. A survey of much of the work done by the start of the 1970s is given in [1, 2]. Questions connected with the motion of material in a liquid state in a screw have been studied the most fully up to the present [1-8]. Among various methods used in practical engineering to calculate the main characteristics of screw extruders, the most accurate and widely used is the method based on a model which realizes the conditions of complex shear [2-4, 6, 8].

In such a statement of the problem (not to mention even simpler ones), however, one is unable to allow for a number of important factors which can have an essential effect on the calculation of the flow characteristics. The point is the ignoring of the influence of the side walls of the channel and the anomaly of the viscosity (except for the power law) in the solution of the problem. But the influence of the walls in the flow of non-Newtonian liquids is either neglected entirely [3, 4], assuming in advance a large ratio W/H, or, as is suggested in [2], it is allowed for through a form factor F_d [5], taken as equal to the value obtained in the integration of the equations of motion of Newtonian liquids. The latter assumption is obviously correct only for materials whose rheological properties differ insignificantly from those of Newtonian meterials [1].

1. Let us consider the steady flow of an incompressible anomalously viscous liquid, assuming that the dimensions of the channel are constant along the length. Moreover, we will assume that the height of the channel is far less than its radius, the curvature of the channel can be neglected, and the gap between the frame and the screw ribs is negligibly small. Then the flow in a screw extruder (Fig. 1a) can be represented as flow in a rectangular channel (Fig. 1b), which develops as a consequence of the motion of the upper plate and the action of counterpressure. In this case the motion and heat exchange of the material are described by a system of differential equations including the equations of motion, incompressibility, and energy and the physical equations. This system must be closed by boundary conditions and by additional equations for the thermophysical properties. In the Cartesian coordinate system the above-indicated equations have the form

$$\rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}, \tag{1}$$

$$\frac{\partial v_i}{\partial x_i} = 0, \tag{2}$$

$$\rho C v_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) + \tau_{ij}\frac{\partial v_i}{\partial x_j}, \tag{3}$$

$$\tau_{ij} = \eta\left(\frac{I_2}{2}\right) f(T)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \tag{4}$$

In the general case the solution of this system presents a complicated problem, even when modern calculating methods and computers are used. It can be simplified considerably, however, if one assumes that the temperature is the same at all points of a channel cross section:

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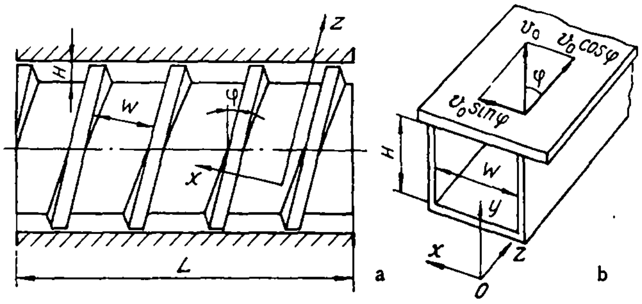

Fig. 1. Schematic drawing of a screw extruder (a) and its plane model (b).

$$\frac{\partial T}{\partial x} = 0, \; \frac{\partial T}{\partial y} = 0. \tag{5}$$

In this case, as shown in [2, 8], the velocity fields do not vary along the longitudinal axis of the channel, i.e., in Eqs. (1-4) the derivatives of the velocity with respect to the z coordinate are equal to zero. The influence of the temperature is reduced to variation only in the stresses, and hence in the pressure and power. And this influence can be estimated through the function f(T) of the temperature dependence of the viscosity, which is equal to one for isothermal flow.

Thus, the solution of the problem of nonisothermal flow is reduced to the solution of two successive problems. First, assigning the determining value of the bulk flow rate Q and assuming that the geometrical dimensions of the extruder, the rotation velocity of the screw, and the rheological properties of the material are known, from the solution of the equations (1), (2), and (4) for isothermal flow we find the velocity fields corresponding to the given value of Q. Then from the known velocities, thermophysical properties, and boundary conditions of heat exchange we determine the temperature distribution along the length of the channel from the energy equation, and knowing this, we calculate the stresses, pressure, and power.

2. Let us briefly discuss the solution of the problem of isothermal flow. Since in actual screws the length of the channel is far greater than its other two dimensions, the pressure along the longitudinal z axis varies by a linear law, i.e., the pressure gradient is $dP/dz = A_1 = \text{const}$. Then it is obvious that the assignment of the flow rate Q is identical to the assignment of the determining value of A_1.

To solve the system of equations describing the two-dimensional flow in a screw, we used the finite-difference method (grid method), which is well recommended in fluid mechanics. Without dwelling in detail on this method, a description of which can be found in many reports (a review is given in [9]), we only note that the equations of motion were first transformed and written through the stream function, the vorticity, and the longitudinal velocity [9, 10]. These transformations and the introduction of the stream function and the vorticity made it possible to eliminate the pressure from the equations of motion and to automatically satisfy the condition of incompressibility.

The boundary values for the new variables were determined from the condition of adhesion of the liquid to the impermeable walls [9]. To solve the system of algebraic equations obtained through the replacement of the differential equations by their finite-difference analogs we used the Gauss–Seidel method of successive approximations. The calculations were performed on a BÉSM-6 computer.

The effectiveness of the use of the finite-difference method to solve such problems and the adequacy of the model of isothermal extrusion described above to the actual process are discussed in [7].

To estimate the influence of the inertial terms and the side ribs on the velocity components of a liquid whose viscosity is described by a power law of flow

$$\eta\left(\frac{I_2}{2}\right) = \eta_0\left(\frac{I_2}{2}\right)^{\frac{n-1}{2}} \tag{6}$$

we carried out a calculation with the following values of the parameters: H = 0.0075 m; W = 0.023 m; L = 0.22 m; φ = 23.5°; v_0 = 12.8 m/sec; n = 0.5; η_0 = 94 $N \cdot sec^{-0.5}/m^2$; $A_1 = 6 \cdot 10^5$ N/m^3; Re = 71. The Reynolds number is computed for the point located at the center of the upper plate.

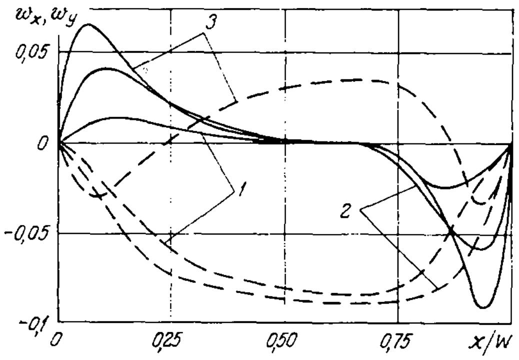

Fig. 2

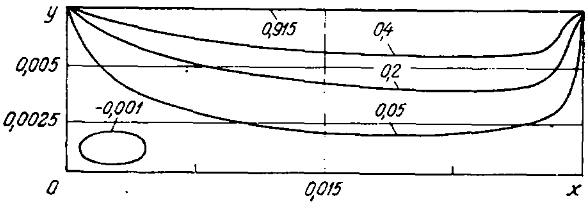

Fig. 3

Fig. 2. Profiles of the dimensionless velocities w_y and w_z in different channel cross sections; Re = 71; solid curves) vertical w_y; dashed curves) transverse w_x; 1) y/H = 0.25; 2) 0.5; 3) 0.75.

Fig. 3. Distribution of dimensionless profile of longitudinal velocity w_z over channel cross section; Re = 71 (x, y, m).

A uniform 17 × 17 grid was used in the calculation. The time for solving the problem was 18.6 min. The results of the calculation are presented in Figs. 2 and 3 in the form of the distribution over the channel cross section of the dimensionless velocities w_x, w_y, and w_z, equal to the ratios of the corresponding components of the true velocities to the velocity v_0.

The velocity profiles are asymmetrical relative to the vertical plane x = W/2 (Figs. 2 and 3), with this asymmetry increasing with an increase in the Reynolds number, as the calculation showed. For large numbers an interesting effect is revealed, consisting in the fact that when a certain value of the counteracting pressure gradient A_1 is reached a region of negative velocities w_z (Fig. 3) is created only in one lower corner, and not in two corners at once, as occurs for small Re [10]. With an increase in A_1 such a region also develops in the other lower corner. With a further increase in A_1 this region gradually covers the entire lower part of the channel.

The presence of inflection points on the velocity profiles together with the asymmetry of the velocity field can, under certain conditions, lead to the appearance of a flow instability, but in the present report we do not consider questions of hydrodynamic instability, the investigation of which is a complicated and independent problem.

The retarding action of the walls on the velocity is seen from the graphs presented. On the whole it agrees with the concepts existing in the literature [1, 2]. It should be noted, however, that the vertical velocity component, which is usually neglected in calculations, becomes comparable with the other two velocities for small W/H, not only near the walls but also at a considerable distance from them (Fig. 2). Therefore, for screws having small ratios W/H the velocity w_y must be allowed for in calculations.

3. After the determination of the velocity field we consider the energy equation. Neglecting the heat conduction along the longitudinal axis of the channel and allowing for the condition (5), we obtain the energy equation in the form

$$\rho C v_z \frac{\partial T}{\partial z} = f(T)\,\eta\left(\frac{I_2}{2}\right)\frac{I_2}{2}. \tag{7}$$

Assuming that the heat exchange with the surrounding medium takes place by Newton's law, after integration we obtain the following transcendental equation for the determination of the temperature at any point of the channel:

$$\int_{T_0}^{T} \frac{C(T)\,dT}{f(T)\left[1 - \frac{\alpha_1 W(T_1 - T) + \alpha_2 W(T_2 - T)}{qf(T)}\right]} = \frac{qz}{\rho Q}. \tag{8}$$

The amount of heat released in the isothermal flow in a section of the channel with a length dz is

$$q = \int_0^W\int_0^H \eta\left(\frac{I_2}{2}\right)\frac{I_2}{2}\,dx\,dy. \tag{9}$$

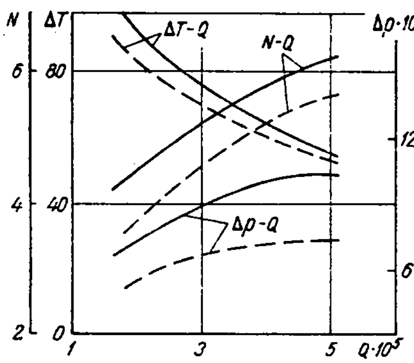

Fig. 4. Dependence of required power and of pressure and temperature drops on liquid flow rate: solid curves) calculation; dashed curves) measurement; N, kW; ΔP, N/m²; ΔT, °K; Q, m³/sec.

The power required by an extruder and the pressure drop in the screw channel are determined from the following equations:

$$N = \int_0^S f(T) \int_0^W [(\tau_{yz} v_z|_{y=H} + (\tau_{xy} v_x)|_{y=H}]\, dxdz \left(S = \frac{L}{\sin\varphi}\right), \tag{10}$$

$$\Delta P = \int_0^S \frac{\partial P}{\partial z} dz = \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) \int_0^S f(T)\, dz - \rho S \left(v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y}\right). \tag{11}$$

In these expressions the shear stresses and velocities are calculated from the equations for isothermal flow. In the case of slow flow Eq. (11) takes the simple form

$$\Delta P = A_1 \int_0^S f(T)\, dz. \tag{12}$$

4. In order to clarify the suitability of the proposed extrusion model for the description of a real process the results of the calculation were compared with experimental data obtained with flow in a screw-type pump of a solution of acrylonitrile copolymer (21% concentration) in dimethyl formamide. The experiments were conducted under conditions close to an adiabatic mode ($\alpha_1 = \alpha_2 = 0$). The dimensions of the screw were the same as in the example considered above. The viscosity of the solution was described by a power law (6) of flow, while the temperature dependences of the specific heat and viscosity were described by the following functions:

$$C(T) = C_0 + C_1(T - T_H); \quad f(T) = \exp[-\beta(T - T_H)].$$

The values of the remaining parameters are $v_0 = 6.28$ m/sec; $n = 0.5$; $\eta_0 = 280\ \text{N}\cdot\text{sec}^{-0.5}/\text{m}^2$; $\rho = 980\ \text{kg/m}^3$; $T_H = 273°\text{K}$; $T_0 = 293°\text{K}$; $C_0 = 1800$ J/kg·deg; $C_1 = 8.4\ \text{J/kg}\cdot\text{deg}^2$; $\beta = 0.015\ \text{deg}^{-1}$.

Curves of the required power and the temperature and pressure drops as functions of the bulk flow rate are presented in Fig. 4. It should be noted first of all that all the curves have the same qualitative character. The slight excess of the theoretical over the experimental heating can be explained by the assumption that the process is adiabatic. The maximum error in the temperature drop does not exceed 12% in the investigated range of flow rates. With an increase in the flow rate the calculated curves of the power and the temperature drop have a tendency to approach the experimental curves, especially the curves of the temperature drop. The minimum errors are 9 to 6%, respectively.

The excess of the calculated over the experimental pressure drop (the maximum is 40% and the minimum is 33%) is evidently connected with liquid leaks in the gap between the screw ribs and the frame. The flow of the leaks, which is not taken into account in the model used, leads to a pressure decrease at the exit from the screw. A particularly strong decrease is observed in modes close to the mode of a closed exit [4]. And since the experiments were conducted in just such modes (the measured flow rate was only a few percent of the maximum flow rate), one can report that after the inflection point on the $\Delta P-Q$ curve the disagreement between the theoretical and experimental data will be considerably smaller as the flow rate increases.

NOTATION

x, y, z, x_i, x_j	are Cartesian coordinates;
H	is the height of channel;
W	is the width of channel;
L and S	are the lengths of the screw and of the channel;
φ	is the pitch of the helical line;
v_0	is the velocity of the upper plate; i, j = 1, 2, 3;
w_x, w_y, w_z	are the dimensionless velocities of the liquid;
v_x, v_y, v_z, v_i, v_j	are the true velocities of liquid particles;
A_1	is the pressure gradient;
Q	is the bulk flow rate of the product;
P	is the pressure;
T	is the temperature;
N	is the power;
τ_{ij}	are the components of the stress tensor;
I_2	is the second (quadratic) invariant of the tensor of deformation velocities;
C and λ	are the specific heat and thermal conductivity of the liquid;
ρ	is the density of the liquid;
α_1 and α_2	are the heat-exchange coefficients;
C_0 and C_1	are the thermophysical constants;
η_0, n, β, T_H	are rheological constants;
T_0	is the liquid temperature at the screw inlet;
Re	is the Reynolds number.

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